Combinatorial Analysis of Casino Games to Unveil Winning Opportunities

William Glory Henderson - 13522113[1]
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
[1]*13522113@std.stei.itb.ac.id*

***Abstract*— This research conducts a comprehensive combinatorial analysis of casino games, including slot machines, blackjack, and poker, with the aim of unveiling potential winning opportunities for players. The study delves into the intricate mathematical and combinatorial aspects of these games, examining probabilities, combinations, and strategies. By dissecting the underlying structures of poker, slot machines, and blackjack, this research seeks to provide valuable insights into optimizing gameplay for casino enthusiasts.**

***Keywords*— Combinatorial analysis, casino games, slot machines, blackjack, poker, winning opportunities, probabilities**

I. INTRODUCTION

Gambling refers to the act of participating in games of chance or betting on uncertain outcomes with the expectation of winning monetary or material rewards. It typically involves risking money or valuables on an event or activity whose result is determined by chance rather than skill, knowledge, or effort. Common forms of gambling include casino games, sports betting, lottery draws, and online gaming. The allure of gambling often lies in the excitement of uncertainty, with participants accepting the risk of losing in the hopes of gaining a financial windfall. While some people engage in gambling as a form of entertainment, it is important to recognize the potential for addiction and negative financial consequences associated with excessive or problematic gambling behavior.[6]

Many believe gambling is designed to favor the house or the entity facilitating the games, while the players will be more likely to lose than win. Casinos and other gambling establishments implement carefully calculated odds and house edges to guarantee their profitability. Due to its inherent difficulty in achieving success, people tend to call gambling “the game of luck”. Nevertheless, gambling is undeniably a mind game that requires a strategic and calculated approach, making it more than just a game of chance. Gambling often relies on random chances and probability. Despite the commonly held perception that these games are solely governed by luck, the reality is that the probability of success can be subjected to mathematical calculations.

The primary objective of this paper is to undertake a comprehensive analysis of the potential for success in casino games through the application of combinatorial principles. The author mainly focuses on discussing the games of blackjack, poker, and slots to examine the various combinations and permutations inherent in these games, with the aim of understanding the factors that contribute to winning outcomes.

II. THEORETICAL FOUNDATION

In contemporary times, the realm of gambling extends beyond traditional casino establishments to encompass online gaming platforms. In contrast to casino games where the odds can be calculated, online gambling introduces an element of uncertainty, as game masters may potentially manipulate the outcomes. This paper, as has been written, will narrow its focus to three specific casino games: blackjack, poker, and slot machines.

2.1. Slot Machine

A slot machine is a gambling device that operates when coins or tokens are inserted into a slot, and a handle is pulled, or a button is pushed to activate reels marked with symbols. These reels are divided into horizontal segments, and the machine pays out by dropping a variable number of coins into a cup or trough based on the alignment of symbols when the reels come to a stop. The goal of slot machines is to align these symbols in specific combinations to win prizes. Common symbols include stars, card suits, bars, numbers (with 7 being popular), various fruits like cherries, plums, oranges, lemons, as well as the words "jackpot" and "bar."[5]

Fig 2.1.1. Slot Machine
Source: https://www.casinonewsdaily.com/slots-guide/types-slot-machines/

In a slot machine, the anticipation and thrill arise from the various combinations of symbol alignments on the spinning reels, each carrying its own distinct winning prize. The machine's paytable outlines the corresponding prizes for different combinations, creating an enticing matrix of potential rewards. Whether it's a trio of matching symbols on a single payline or a more intricate arrangement across multiple lines, each alignment corresponds to a unique winning outcome.

2.2. Blackjack

Blackjack, also known as twenty-one, is a card game where the goal is to receive cards with a total value higher than the dealer's, but not surpassing 21. The dealer can use a single deck of 52 cards or multiple decks from a container known as a shoe. Aces can be counted as either 1 or 11, and face cards have a value of 10.[1]

Fig 2.2.1. Blackjack
Source: https://www.stat.colostate.edu/~meyer/blackjack.htm

The primary goal in blackjack is to approach the value of 21 without exceeding it, all while having a hand that beats the dealer's. Achieving this involves totaling the card values in your hand. While single and double deck games exist, the most prevalent format involves a 6-deck or 8-deck shoe. Cards numbered 2 through 10 hold their face value, face cards are valued at 10, and an ace can be valued at either 1 or 11 at the player's choice. Hands with an ace valued at 11 are considered 'soft' (e.g., A, 6 is a soft 17), while those with an ace valued at 1 are 'hard' (e.g., A, 6, Q is a hard 17). A blackjack hand consists of an ace and a 10-valued card (10, J, Q, K). A two-card blackjack triumphs over three or more cards totaling 21. Before receiving any cards, players must place a wager, after which they are dealt 2 cards face up. The dealer also receives two cards, one face up and the other face down. The dealer's 'hole' card remains concealed until players complete their hands. The dealer then reveals the hole card, standing on a hard 17 through 21 and hitting on a soft 16. If the dealer's hand is 16 or under, they must continue drawing cards until exceeding 16 or going over 21 (bust).

In addition, if a player's cards exceed 21 (busted), the player loses; if the dealer's cards exceed 21, the player wins. If the dealer's cards have the same value as the player's cards, the game ends in a tie. Players can also win by having five cards without the total value exceeding 21.

Beyond these fundamental steps, players can employ advanced tactics like doubling down, splitting pairs, or surrendering to enhance their chances of success in this dynamic and strategic card game.

2.3. Poker

The goal of poker is to either have the best hand at the showdown or convince other players to fold, thereby winning the pot. Poker games typically involve betting rounds, where players can check, bet, raise, or fold, adding an element of psychological gameplay. Key elements include community cards, shared by all players, and the use of various hand rankings to determine the winner.

Here is a brief explanation about the player's hand cards, sorted from the strongest to the wakest. Poker Hand Rankings:[3]

a. Royal Flush

A royal flush consists of A, K, Q, J and 10, all the same suits.

Fig 2.3.1. Royal Flush
Source: https://upswingpoker.com/poker-rules/

b. Straight Flush

A straight flush consists of any straight value that is all the same suits.

Fig 2.3.2. Straight Flush
Source: https://upswingpoker.com/poker-rules/

c. Four of a Kind

Four of a kind or ‘quads’ consists of four cards of equal value along with another card known as a side card.

Fig 2.3.3. Four of a Kind
Source: https://upswingpoker.com/poker-rules/

d. Full House

A full house consists of three cards of one value and two cards of another.

Fig 2.3.4. Full House
Source: https://upswingpoker.com/poker-rules/

e. Flush

A flush is a hand which has all cards of the same suit.

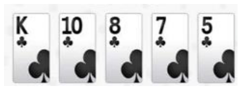

Fig 2.3.5. Flush
Source: https://upswingpoker.com/poker-rules/

f. Straight

A straight has 5 cards of consecutive value that are not all the same suits.

Fig 2.3.6. Straight
Source: https://upswingpoker.com/poker-rules/

g. Three of a Kind

Also known as ‘trips’, three of a kind is 3 cards of the same value and 2 side cards of different value.

Fig 2.3.7. Three of a Kind
Source: https://upswingpoker.com/poker-rules/

h. Two Pair

Two pairs consist of two cards of equal value, another two cards of equal value, and one extra card.

Fig 2.3.8. Two Pair

Source: https://upswingpoker.com/poker-rules/

i. Pair

One pair consists of two cards of the same value, and three extra cards.

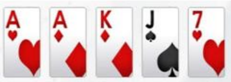

Fig 2.3.9. Pair

Source: https://upswingpoker.com/poker-rules/

j. High Card

Five cards that do not interact with each other to make any of the above cards.

Fig 2.3.10. High

Source: https://upswingpoker.com/poker-rules/

III. PROBABILITY AND COMBINATORICS

3.1. Probability

Probability refers to the likelihood or possibility of an event occurring and is a mathematical discipline that focuses on random events. Its values range from zero to one. Probability is integrated into mathematics to forecast the likelihood of events. Essentially, it signifies the extent to which something is expected to happen. This fundamental concept of probability is also applied in probability distribution, where the likelihood of outcomes in a random experiment is explored. To determine the probability of a singular event, one must initially determine the total number of potential outcomes.

In probability theory, combinatorics finds various applications. To illustrate, when determining the probability of a specific event A, the following formula can be employed:[4]

$$P(A) = Probability\ of\ event\ A = \frac{Frequency\ of\ A}{Total\ Sample\ Space} \quad (1)$$

$P(A)$ represents the probability of event A occurring. The numerator is the count of outcomes that lead to the occurrence of event A, and the denominator is the total number of possible outcomes (sample space).

Key concept in probability:

1. **Sample Space:** The set of all possible outcomes of an experiment is called the sample space, denoted by S.
2. **Event:** An event is a subset of the sample space, representing a specific outcome or a combination of outcomes.
3. **Probability of Complementary Events:** The probability of an event A happening is denoted by $P(A)$, and the probability of the complement of A (not A happening) is denoted by $P(A^-)$, where;

$$P(A^-) = 1 - P(A) \quad (2)$$

4. **Addition Rule:** The probability of the union of two events A and B is given by $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $P(A \cap B)$ is the probability of both events occurring.
5. **The Ranges:** $0 \le p(x) \le 1$ or $0\% \le p(x) \le 100\%$

3.2. Combinatorics

Combinatorics is a branch of mathematics focused on the examination of finite discrete structures. It involves the study of permutations and combinations, as well as the enumeration of sets of elements, elucidating mathematical relations and their properties. The term "Combinatorics" is commonly used by mathematicians to denote a broader category within Discrete Mathematics. This field is extensively employed in computer science for deriving formulas and analyzing algorithms. This article will delve into the intricacies of combinatorics, exploring its characteristics, formulas, applications, and providing detailed examples.[2]

3.2.1. Multiplication rule

It states that if there are n_1 ways to perform the first operation, and for each of those ways, there are n_2 ways to perform the second operation, and so on, then the total number of ways to perform the entire sequence of events is the product of n_1, n_2, ….

Mathematically, if there are k events in a sequence, and the number of ways to perform the i-th event is n_i, then the total number of outcomes N is given by:

$$N = n_1 \times n_2 \times \ldots \times n_k \quad (3)$$

For example, consider the following scenario: if you have m choices for the first event and, independently, n choices for the second event, then the total number of ways to perform both events is $m \times n$.

3.2.2. Addition rule

The addition rule in combinatorics is another fundamental principle used to calculate the total number of outcomes for a set of events that can occur separately or mutually exclusively. This rule states that if there are n_1 ways to perform the first event, and there are n_2 ways to perform the second event, and so on, then the total number of ways to perform at least one of these events is the sum of n_1, n_2, ….

Mathematically, if there are k events, and the number of ways to perform the i-th event is ni, then the total number of outcomes N is given by:

$$N = n_1 + n_2 + \cdots + n_k \quad (4)$$

For example, consider the scenario where you have m choices for one event and n choices for another event, and you want to calculate the total number of ways to perform either the first or the second event. In this case, the total number of outcomes is $m + n$.

3.2.3. Permutation

A permutation refers to an arrangement of objects in a specific order. The number of permutations of n distinct objects taken r at a time is given by:

$$_{n}P_{r} = \frac{n!}{(n-r)!} \quad (5)$$

For example, if there are three distinct objects (A, B, C), the permutations of two taken at a time would be AB, BA, AC, CA, BC, CB.

3.2.4. Combination

A combination is a selection of objects without regard to the order in which they are arranged. The number of combinations of n distinct objects taken r at a time is given by:

$$_{n}C_{r} = \frac{n!}{r!(n-r)!} \quad (6)$$

For example, if there is the same set of three distinct objects (A, B, C), the combinations of two taken at a time would be AB, AC, and BC.

IV. ANALYSIS & IMPLEMENTATION

4.1. Slot Machine

Every slot machine has a different probability of winning the game. Each combination of images or symbols leads to different prizes or cash rewards.

Fig 4.1. Slot Machine Screen
Source: Red, White, and Blue Slot Review: Classic Vegas Casual Game (pacasino.com)

Table 4.1.1. Slot Machine Paytable[5]

Combinations of Symbols			Rewards Multiply
Red 7	White 7	Blue 7	2500
Red 7	Red 7	Red 7	250
White 7	White 7	White 7	200
Blue 7	Blue 7	Blue 7	150
Any 7	Any 7	Any 7	80
1 Bar	2 Bar	3 Bar	51
3 Bar	3 Bar	3 Bar	40
2 Bar	2 Bar	2 Bar	25
Any red	Any white	Any blue	20
1 Bar	1 Bar	1 Bar	10
Any bar	Any bar	Any bar	5
Any red	Any red	Any red	2
Any white	Any white	Any white	2
Any blue	Any blue	Any blue	2
Any blank	Any blank	Any blank	1

In slot machine, bar 1 is red, bar 2 is white, and bar 3 is blue. The position of the symbol's combinations must be aligned with the rewards lists; reel 1 for column 1, reel 2 for column 2, and reel 3 for column 3. "Any red" indicates the row contains any kind of red symbols (bar 1 and red 7). Bar 2 is white and bar 3 is blue. Rewards multiply comes from player's bet, if the player bet 1000 and the player got red, white, and blue it multiplies into 2500 x 1000. So, the total reward cash is player's bet x reward multiply.

If the player gets the combination of Red 7, White 7, and Blue 7, which is included to both (Red 7, White 7, Blue 7) and (Any 7, Any 7, Any 7) Rewards Multiply, so the player will be getting the (Red 7, White 7, Blue 7) Rewards Multiply, because "Any" works when it is not included to the other combination of symbols. This rule also applies to all combinations of "Any".

Table 4.1.2. Frequency of Symbols Appearance in Slot Machine

Symbol	Reel 1	Reel 2	Reel 3
Red 7	1	3	1
White 7	6	1	7
Blue 7	6	7	1
3 Bar	6	7	5
2 Bar	7	6	9
1 Bar	6	8	9
Blank	32	32	32
Total	64	64	64

From the table, each reel column represents the frequency of occurrence of symbols. The total symbols from all reels are 64 + 64 + 64 = 192. Also, that the sample space are 64 x 64 x 64 = 262144.

The probability formula is the number of symbols appearing on the reels then divided by the total symbols on the reel, which is 64. We are looking for the probability according to the paytable.

Probability for Red 7, White 7, and Blue 7 :

$$\frac{1}{64} \times \frac{1}{64} \times \frac{1}{64} = \frac{1}{262144}$$

Probability for Red 7, Red 7, and Red 7 :

$$\frac{1}{64} \times \frac{3}{64} \times \frac{1}{64} = \frac{3}{262144}$$

Probability for White 7, White 7, and White 7 :

$$\frac{6}{64} \times \frac{1}{64} \times \frac{7}{64} = \frac{42}{262144}$$

Probability for Blue 7, Blue 7, and Blue 7 :

$$\frac{6}{64} \times \frac{7}{64} \times \frac{1}{64} = \frac{42}{262144}$$

Probability for Any 7, Any 7, and Any 7 :

$$\frac{13}{64} \times \frac{11}{64} \times \frac{9}{64} - \frac{1+3+42+42}{262144} = \frac{1199}{262144}$$

Probability for 1 Bar, 2 Bar, and 3 Bar :

$$\frac{6}{64} \times \frac{6}{64} \times \frac{5}{64} = \frac{180}{262144}$$

Probability for 3 Bar, 3 Bar, and 3 Bar :

$$\frac{6}{64} \times \frac{7}{64} \times \frac{5}{64} = \frac{210}{262144}$$

Probability for 2 Bar, 2 Bar, and 2 Bar :

$$\frac{7}{64} \times \frac{6}{64} \times \frac{9}{64} = \frac{378}{262144}$$

Probability for Any Red, Any White, and Any Blue:

$$\frac{7}{64} \times \frac{9}{64} \times \frac{6}{64} - \frac{1+180}{262144} = \frac{113}{262144}$$

Probability for 1 Bar, 1 Bar, and 1 Bar :

$$\frac{6}{64} \times \frac{8}{64} \times \frac{9}{64} = \frac{432}{262144}$$

Probability for Any Bar, Any Bar, and Any Bar:

$$\frac{19}{64} \times \frac{21}{64} \times \frac{23}{64} - \frac{210+180+378+432}{262144} = \frac{7977}{262144}$$

Probability for Any Bar, Any Bar, and Any Bar:

$$\frac{19}{64} \times \frac{21}{64} \times \frac{23}{64} - \frac{210+180+378+432}{262144} = \frac{7977}{262144}$$

Probability for Any Red, Any Red, and Any Red:

$$\frac{7}{64} \times \frac{11}{64} \times \frac{10}{64} - \frac{3+432}{262144} = \frac{335}{262144}$$

Probability for Any White, Any White, and Any White:

$$\frac{13}{64} \times \frac{7}{64} \times \frac{16}{64} - \frac{42+378}{262144} = \frac{1036}{262144}$$

Probability for Any Blue, Any Blue, and Any Blue :

$$\frac{12}{64} \times \frac{14}{64} \times \frac{6}{64} - \frac{42+210}{262144} = \frac{756}{262144}$$

Probability for Blank, Blank, and Blank :

$$\frac{32}{64} \times \frac{32}{64} \times \frac{32}{64} = \frac{32768}{262144}$$

From all the calculation, we can determine the total chance of winning the game is the sum of all the probabilities, that is $\frac{45472}{262144}$ or about 17,35 %. By using the probability property

$$P(A^{-}) = 1 - P(A) \quad (2)$$

P(A⁻) represents the chance of losing and P(A) represents the chance of winning the game. The chance of a player losing the game is $\frac{216672}{262144}$ or around 82,65%. So, players will be more likely to experience losses than profits.

4.2. Blackjack

In casinos, blackjack games commonly use 6 or 8 decks of cards. In this case, using 6 decks results in a total of 6 x 52 = 312 cards. Each deck has 4 cards with the same rank (9♣ 9♠ 9♦ 9♥) so the total each card on 6 decks is 24.

$$P(A) = \frac{\textit{Number of ways to choose from each card rank}}{\textit{Number of ways to choose 2 from 312}}$$

The player will be able to count the probabilities of 2 aces by using the rules.

$$\frac{{}_{24}C_2}{{}_{312}C_2} = 0.57\ \%$$

24 is the total aces and 2 is the number of cards that taken

To get the probability number of 21 (A, (10, J, Q, K)), we can implement this formula:

$$\frac{{}_{24}C_1 \times {}_{96}C_1}{{}_{312}C_2} = 4.75\ \%$$

The total each card on 6 decks is 24. Because there are 4 cards with the same value of 10, so the total is 24 x 4 = 96. The above formula will also apply to the following combinations of cards.

Table 4.2.1. Card Combinations Probability Table

Card Combinations	Value	Probability
A, (J, Q, K, or10)	21	4.75 %
(10, J, Q, or K), (10, J, Q, or K)	20	9.40 %
9, A		1.19 %
9. (10, J, Q, or K)	19	4.75 %
8, A		1.19 %
9, 9	18	0.57 %
8, (10, J, Q, or K)		4.75 %
7, A		1.19 %
8, 9	17	1.19 %
7, (10, J, Q, or K)		4.75 %
6, A		1.19 %
8, 8	16	0.57 %
7, 9		1.19 %
6, (10, J, Q, or K)		4.75 %
5, A		1.19 %
7, 8	15	1.19 %
6, 9		1.19 %
5, (10, J, Q, or K)		4.75 %
4, A		1.19 %

7, 7	14	0.57 %
6, 8		1.19 %
5, 9		1.19 %
4, (10, J, Q, or K)		4.75 %
3, A		1.19 %
6, 7	13	1.19 %
5, 8		1.19 %
4, 9		1.19 %
3, (10, J, Q, or K)		4.75 %
2, A		1.19 %
6, 6	12	0.57 %
5, 7		1.19 %
4, 8		1.19%
3, 9		1.19 %
2, (10, J, Q, or K)		4.75 %
A, A		0.57%
A, (10, J, Q, or K)	11	4.75 %
2, 9		1.19 %
3, 8		1.19 %
4, 7		1.19 %
5, 6		1.19 %
A, 9	10	1.19 %
2, 8		1.19 %
3, 7		1.19 %
4, 6		1.19 %
5, 5		0.57 %
A, 8	9	1.19 %
2, 7		1.19 %
3, 6		1.19 %
4, 5		1.19 %
A, 7	8	1.19 %
2, 6		1.19 %
3, 5		1.19 %
4, 4		0.57 %
A, 6	7	1.19 %
2, 5		1.19 %
3, 4		1.19 %
A, 5	6	1.19 %
2, 4		1.19 %
3, 3		0.57 %
A, 4	5	1.19 %
2, 3		1.19 %
A, 3	4	1.19 %
2, 2		0.57 %
A, 2	3	1.19%
A, A	2	0.57%

The total probability of all combinations is more than 100% due to the presence of an ace card, which can have two values (1 and 11). Therefore, we can choose whether to consider the ace as 1 or 11.

If the player does not get blackjack when receiving two initial cards, the player can choose to hit (taking a card) or stay but most players will hit when the total value is below 12, in order to increase its value. Therefore, it is important for player to know the card probability when the total value reach more than 21 or busted.

Table 4.2.2. The probability of cards having a value greater than 21 after hit

Card Combinations in Hand	Value	Probability
A, (J, Q, K, or10)	21	100 %
(10, J, Q, or K), (10, J, Q, or K)	20	92.3 %
9, A		0 %
9. (10, J, Q, or K)	19	84.6 %
8, A		0 %
9, 9	18	76.9 %
8, (10, J, Q, or K)		76.9 %
7, A		0 %
8, 9	17	69.2 %
7, (10, J, Q, or K)		69.2 %
6, A		0 %
8, 8	16	61.5 %
7, 9		61.5 %
6, (10, J, Q, or K)		61.5 %
5, A		0 %
7, 8	15	53.9 %
6, 9		53.9 %
5, (10, J, Q, or K)		53.9 %
4, A		0 %
7, 7	14	46.2 %
6, 8		46.2 %
5, 9		46.2 %
4, (10, J, Q, or K)		46.2 %
3, A		0 %
6, 7	13	38.5 %
5, 8		38.5 %
4, 9		38.5 %
3, (10, J, Q, or K)		38.5 %
2, A		0 %
6, 6	12	30.8 %
5, 7		30.8 %
4, 8		30.8%
3, 9		30.8 %
2, (10, J, Q, or K)		30.8 %
A, A		0 %

For cards with an initial value of 11 or below, hitting it 1x will not exceed 21 so the percentage of getting busted is 0. In the case of an Ace, its percentage also 0 because an Ace can have a value of either 1 or 11.

Table 4.2.3. Probability of winning in Blackjack from 2 Initial Cards

Card Value (player)	Win	Lose	Draw
21	50 %	0 %	50 %
20	40 %	20 %	40 %
19	30 %	40 %	30 %
18	20 %	60 %	20 %
17	10 %	80 %	10 %

Table 4.2.4. Probability of winning in Blackjack After Hit 1 Card

Card Value (player)	Win	Lose	Draw
21	50 %	0 %	50 %

20	41.7 %	16.6 %	41.7 %
19	33.3 %	33.4 %	33.3 %
18	25 %	50 %	25 %
17	16.7 %	66.6 %	16.7 %

If the dealer gets 21 from 2 initial cards then the dealer wins the game or draws if the player also gets 21 from 2 initial cards.

Since the dealer has to be 17 or above, the probability of winning the game if the player card under 17 is 16.7 %. It is because the player only can win if the dealer gets busted. The calculation can be seen in the appendix.

From both calculations, the overall probability of the game is (the calculation can be seen in the appendix)

$$win = 31.7\ \%$$
$$lose = 36.6\ \%$$
$$draw = 31.7\%$$

4.3. Poker

In casinos, poker commonly use 1 deck of card. 1 deck of card consists of 52 cards and 1 poker hand ranking consists of 5 cards. So, the total card combination of poker can be mathematically solved by using this formula:

$$_{52}C_5 = 2598960$$

Royal Flush

For all types, there is only 1 possibility of royal flush. So, there are 4 total of probabilities.

$$\frac{4}{2598960} = 0.000154\ \%$$

Straight Flush

In straight flush, there are 9 possibilities (Ace – 9) for each suit. So, the total is 9 x 4 = 36 probabilities.

$$\frac{36}{2598960} = 0.00139\ \%$$

Four of a Kind

Because the other card is random, there are 13 possibilities for the 4 same cards. So, there are 48 probabilities and 13 x 48 = 624 probabilities in total.

$$\frac{624}{2598960} = 0.024\ \%$$

Full House

Three of a Kind means we take 3 from 4 cards. There are 13 rank possible card. For the other One Pair, that means we take 2 from 4 cards. And the rest are 12 possibilities. The total is $_4C_3$ x 13 x $_4C_2$ x 12 = 3744 probabilities in total.

$$\frac{3744}{2598960} = 0.144\ \%$$

Flush

In flush, for each suit, we take 5 from 13 cards (not in order). Then $_{13}C_5$ must be minus by 10 (Straight Flush and Royal Flush), then multiplied by 4. The total is ($_{13}C_5$ – 10) x 4 = 5108 probabilities in total.

$$\frac{5108}{2598960} = 0.197\ \%$$

Straight

There are 10 possible of draws (which start from A-2-3-4-5-10-J-Q-K-As). Each card is free of suit, but they can't all be the same. That means there are 4^5 possible suits minus by 4 (all suits are the same). The total is 10 x (4^5 – 4) = 10200 probabilities in total.

$$\frac{10200}{2598960} = 0.392\ \%$$

Three of a Kind

Three of a Kind means we are taking 3 out of 4 cards, there are 13 choices. The first 3 cards has a probability of $_4C_3$ x 13 = 52 The fourth card has 48 possibilities (cannot be the same as the first 3 cards). Fifth Card has 44 possibilities cannot be the same as the first 3 cards and the fourth card). Because of the fourth card and fifth does not affect the order, so it must be divided by 2!. So the total is 52 x 48 x 44 / 2 = 54912 probabilities in total.

$$\frac{54912}{2598960} = 2.113\ \%$$

Two Pair

There are 2 pairs of cards. The last card cannot be the same as the previous card, so there are 44 possible final cards. We need to choose 2 pairs from 13 existing suits, and each pair has $_4C_2$ possibilities. So the total is $_{13}C_2$ x $_4C_2$ x $_4C_2$ x $_{44}C_1$ = 123552 probabilities in total.

$$\frac{123552}{2598960} = 4.754\ \%$$

Pair

For the 2 same cards, there are $_4C_2$ possibilities and 13 chosen suits. The remaining cards can't form anything, so it's all three must be a different suit. That means we are taking 3 out of 12 cards, and each has 4 possible colors. So, there is $_4C_2$ x 13 x $_{12}C_3$ x 4^3 = 1098240 probabilities in total.

$$\frac{1098240}{2598960} = 42.257\ \%$$

High Card

High card is the five cards that do not interact with each other. So, we can calculate the probability with the probability formula of :

$$P(A^{-}) = 1 - P(A) \qquad (2)$$

P(A^-) represents the probability of High Card and P(A) represents the total of probabilities above. So the probability of the High Card is 50.117 %

Table 4.3.1. Poker probability of winning table

Combination	Win	Lose	Draw
Royal Flush	50 %	0 %	50 %
Straight flush	45 %	10 %	45 %
Four of a Kind	40 %	20 %	40 %
Full House	35 %	30 %	35 %
Flush	30 %	40 %	30 %
Five Straight	25 %	50 %	25 %
Three of a Kind	20 %	60 %	20 %
Two Pair	15 %	70 %	15 %
One Pair	10 %	80 %	10 %
High Card	5 %	90 %	5 %

The winning table probability is based on the card that the player gets. If the player has a straight flush in hand, then the player can only be lose to a full house combination. The winning probability should be 90% but if the other player gets the same straight flush, it would be a draw. So, the probability is divided by 2 because draw does not counted as a lose, it will not disadvantage the player (the bet returns 100% to the player).

The overall probability of the poker game is

$$win = 27.4\ \%$$
$$lose = 45.2\ \%$$
$$draw = 27.4\ \%$$

This game tends to lean towards losses because the chances of getting high cards are around 50 % and the likelihood of losing with high cards is about 90 %. As a result, it is more likely to experience losses than wins. The overall loss calculation is 45.2 % because the chance of a high card appearing is around 50 % which means most of the loss is caused by a high card. So, the loss overall is around half of the high card loss.

V. CONCLUSION

There are several things that can be concluded from the discussion above:

1. Discrete mathematics, particularly combinatorics, offers valuable tools that can be applied to address everyday challenges, exemplified by its utility in solving problems related to gambling.
2. The possibility of a player losing in a slot game indicates that the odds are more towards losses than profits. The probability of winning in slot machine is 17.35% while the probability of losing is 82.65%
3. The likelihood of winning in poker (27.4%) and Blackjack (31.7%) are lower than the likelihood of losing, while the probability of winning and tying in both games are the same.
4. It is better for us to not gambling, but if players want to do gambling, poker and blackjack are better choices than slot machine based on probability.

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REFERENCES

[1] Blackjack summary (no date) Encyclopædia Britannica. Available at: https://www.britannica.com/summary/blackjack-card-game (Accessed: 08 December 2023).

[2] *Combinatorics* (2023) *Encyclopædia Britannica*. Available at: https://www.britannica.com/science/combinatorics (Accessed: 11 December 2023).

[3] How to play poker: Basic poker rules for new players (2023) Upswing Poker. Available at: https://upswingpoker.com/poker-rules/ (Accessed: 08 December 2023).

[4] *Probability in maths - definition, formula, types, problems and solutions, BYJUS*. Available at: https://byjus.com/maths/probability/(Accessed: 08 December 2023).

[5] Shackleford, M. (2019) Red White & Blue Analysis, Wizard Of Odds > Guide to Gambling Games & Online Casinos. Available at: https://wizardofodds.com/games/slots/appendix/6/(Accessed: 09 December 2023).

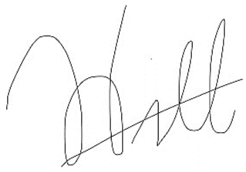
[6] Slot machine (2023) Encyclopædia Britannica. Available at: https://www.britannica.com/topic/slot-machine (Accessed: 8 December 2023). *Blackjack summary* (no date) *Encyclopædia Britannica*. Available at: https://www.britannica.com/summary/blackjack-card-game(Accessed: 08 December 2023).

[7] Ygi (no date) What is gambling? Available at: https://youthgambling.mcgill.ca/Gambling2/en/adolescents/whatisgambling.php (Accessed: 08 December 2023).

PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 11 Desember 2023



William Glory Henderson
13522113

APPENDIX

```python
import math

## find total combination (nCr)
x = 0   # number of n
y = 0   # number of r
result = math.comb(x,y)
print (result)

## find winning probabilities on blackjack
countWin1 = 0
countWin2 = 0
countLose1 = 0
countLose2 = 0
countDraw1 = 0
countDraw2 = 0

# i = The value card on player hand starts from 4 until 21 (from 2 initial cards)
for i in range (18):
    if (i < 13):
        winProb = 1 / 6 * 100
        loseProb = 5 / 6 * 100
        drawProb = 0
        print(f'Win {i + 4} : {winProb}%')
        print(f'Lose {i + 4} : {loseProb}%')
        print(f'Draw {i + 4} : {drawProb}%')
    else:
        winProb = (i - 12) / 10 * 100
        loseProb = 100 - 2 * winProb
        drawProb = (i - 12) / 10 * 100
        countWin1 = winProb + countWin1
        countLose1 = loseProb + countLose1
        countDraw1 = drawProb + countDraw1
        print(f'Win {i + 4} : {winProb}%')
        print(f'Lose {i + 4} : {loseProb}%')
        print(f'Draw {i + 4} : {drawProb}%')

# j = The value card on player hand starts from 4 until 21 (After hit)
for j in range (18):
    if (j < 13):
        winProb = 1 / 6 * 100
        loseProb = 5 / 6 * 100
        drawProb = 0
        print(f'Win {j + 4} : {winProb}%')
        print(f'Lose {j + 4} : {loseProb}%')
        print(f'Draw {j + 4} : {drawProb}%')
    else:
        winProb = (j - 11) / 12 * 100
        loseProb = 100 - 2 * winProb
        drawProb = (j - 11) / 12 * 100
        countWin2 = winProb + countWin2
        countLose2 = loseProb + countLose2
        countDraw2 = drawProb + countDraw2
        print(f'Win {j + 4} : {winProb}%')
        print(f'Lose {j + 4} : {loseProb}%')
        print(f'Draw {j + 4} : {drawProb}%')

# Overall Probabilities
print(f'Win Overall : {(countWin1 + countWin2) / 10}')
print(f'Lose Overall : {(countLose1 + countLose2) / 10}')
print(f'Draw Overall : {(countDraw1 + countDraw2) / 10}')

## find winning probabilities on poker
# k = The card in player hand (0 = royal flush, 1 = straight flush, ..., 9 = high card)
for k in range(10) :
    winProb = (10 - k) / 2 * 10
    loseProb = 100 - 2 * winProb
    drawProb = (10 - k) / 2 * 10
    print(f'Win {k} : {winProb}%')
    print(f'Lose {k} : {loseProb}%')
    print(f'Draw {k} : {drawProb}%')
```